TRINITY COLLEGE

IN NOMINE DOMINI

YEAR 12
MATHEMATICS
METHODS

Test 3, 2023
Section One: Calculator Free
Logs, Log Calculus & CRVs

STUDENT'S NAME: MARKING KEY [KRISZYK]

DATE: Wednesday 2nd August **TIME:** 30 minutes **MARKS:** 33
ASSESSMENT %: 10

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser
Special Items: Formula sheet

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

Question 1 **(7 marks)**

(a) Show that $\log 20 + \log 5 = 2$ (2 marks)

$$\log 20 + \log 5 = \log 100 \quad ✓$$
$$= \log_{10} 10^2 \quad ✓$$
$$= 2$$

must show $100 = 10^2$ or similar

(b) Express $3\log x - (\log y + 3)$ as a single logarithm. (3 marks)

$$= 3\log x - (\log y + 3\log 10) \quad ✓$$
$$= 3\log x - \log 1000y \quad ✓$$
$$= \log\left(\frac{x^3}{1000y}\right) \quad ✓$$

(c) If $\log 5 = a$ and $\log 7 = b$, express $\log 350$ in terms of a and b. (2 marks)

$$\log(350) = \log(5 \times 7 \times 10)$$
$$= \log 5 + \log 7 + \log 10 \quad ✓$$
$$= a + b + 1 \quad ✓$$

Question 2 **(8 marks)**

Differentiate with respect to x.

(a) $f(x) = \ln(2x+1)$ (1 mark)

$f'(x) = \dfrac{2}{2x+1}$ ✓

(b) $f(x) = \ln\left(\dfrac{x^2+2x}{x-5}\right)$ (2 marks)

$f(x) = \ln(x^2+2x) - \ln(x-5)$ ✓

$f'(x) = \dfrac{2x+2}{x^2+2x} - \dfrac{1}{x-5}$ ✓

(c) $f(x) = \dfrac{2\sqrt{x}}{\ln x}$ (3 marks)

u $\quad u' = x^{-1/2}$

v $\quad v' = \frac{1}{x}$

$f'(x) = \dfrac{x^{-1/2}\ln x - \frac{1}{x}(2\sqrt{x})}{(\ln x)^2}$ ✓✓

$= \dfrac{\ln(x) - 2}{\sqrt{x}(\ln x)^2}$ ✓

No need to simplify for 3
just no negative indices

(d) $f(x) = \log_5(x^2)$ (2 marks)

$f(x) = \dfrac{\ln x^2}{\ln 5}$ ✓

$= \dfrac{2x}{x^2 \ln 5}$ or $\dfrac{2}{x\ln 5}$ ✓

Question 3 **(7 marks)**

(a) Determine the following indefinite integrals. Assume denominators are greater than zero.

(i) $\int \frac{12}{x} dx$ (1 mark)

$= 12 \ln x + c$ ✓

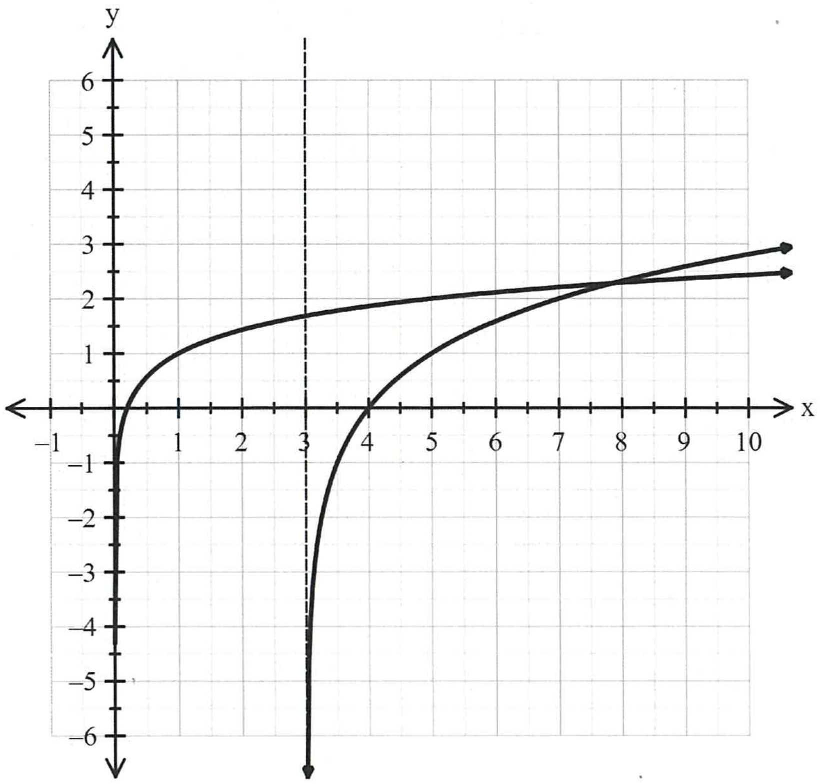
(ii) $\int \frac{\sin 3x}{\cos(3x)+1} dx$ (2 marks)

$f(x) = \cos(3x) + 1$

$f'(x) = -3\sin(3x)$

$= -\frac{1}{3} \ln(\cos(3x) + 1) + c$ ✓✓

(b) The graphs of $f(x)$ and $g(x)$ are drawn below where $f(x) = \log_a(x-b)$ and $g(x) = \log_c(x) + d$. Determine the values of a, b, c and d. (4 marks)



$a = 2$ ✓

$b = 3$ ✓

$c = 5$ ✓

$d = 1$ ✓

Question 4 **(5 marks)**

Solve for the exact value of x in the following equation $\left(3^x\right)\left(4^{2x+1}\right) = 6^{x+2}$.

Give your answer in the form $\dfrac{\ln a}{\ln b}$.

$$\ln\left(3^x \cdot 4^{2x+1}\right) = \ln\left(6^{x+2}\right) \quad ✓$$

$$x\ln 3 + (2x+1)\ln 4 = (x+2)\ln 6 \quad ✓$$

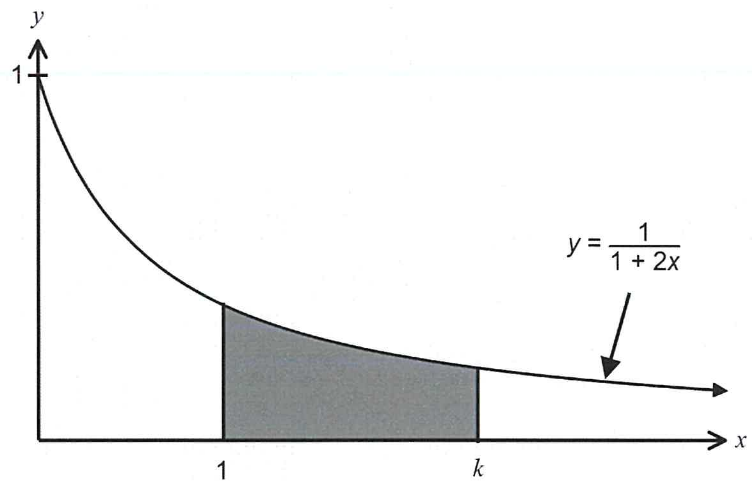
$$x\ln 3 + x\ln 4^2 + \ln 4 = x\ln 6 + \ln 6^2 \quad ✓$$

$$x\left[\ln\left(\frac{3\times 16}{6}\right)\right] = \ln 9 \quad ✓$$

$$x = \frac{\ln 9}{\ln 8} \quad ✓$$

Question 5 **(6 marks)**

In the following diagram the shaded area represents a value of 0.2 square units. Determine the exact value of k.



$$\int_1^k \frac{1}{1+2x}\,dx = 0.2 \quad ✓$$

$$\left[\frac{1}{2}\ln(1+2x)\right]_1^k = 0.2 \quad ✓✓$$

$$\frac{1}{2}\left[\ln(1+2k) - \ln(1+2)\right] = 0.2 \quad ✓$$

$$\ln\left(\frac{1+2k}{3}\right) = 0.4$$

$$e^{0.4} = \frac{1+2k}{3} \quad ✓$$

$$k = \frac{3e^{0.4}-1}{2} \quad ✓$$

END OF QUESTIONS

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Test 3, 2023
Section Two: Calculator Allowed
Logs, Log Calculus & CRVs

STUDENT'S NAME: MARKING KEY [KRISZYK]

DATE: Wednesday 2nd August **TIME:** 20 minutes **MARKS:** ~~22~~ 21
ASSESSMENT %: 5

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser
Special Items: 1 A4 page notes, Classpad, Scientific Calculator

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

Question 6

(~~4~~ 3 marks)

The magnitude of an earthquake on the Richter scale, R, is given by $R = \log(I), I \geq 1$ where I is the relative intensity of the earthquake compared to the smallest seismic activity that can be measured. The location, year and magnitude of three large earthquakes are given in this table.

Earthquake	Meeberrie, 1941	Sumatra, 2005	Valdivia, 1960
Magnitude	7.2	8.6	9.5

(a) Calculate the magnitude of the 1968 earthquake in Meckering, 130 km inland from Perth, which had a relative intensity of 7 940 000. (1 mark)

6.9

(b) The strongest earthquake ever recorded in the world was at Valdivia, Chile, in 1960. The strongest onshore earthquake ever recorded in Australia was at Meeberrie in 1941.

Express the ratio of the relative intensities of the Meeberrie to the Valdivia earthquake in the form 1:n . (2 marks)

$I_m = 10^{7.2} \qquad I_v = 10^{9.5}$

Ratio is $1 : \dfrac{10^{9.5}}{10^{7.2}}$ ✓

$= 1 : 10^{2.3}$

$\sim 1 : 200$ ✓

Question 7 **(11 marks)**

A pizza shop estimates that the time X hours to deliver a pizza from when it is ordered is a continuous random variable with probability density function given by

$$f(x)=\begin{cases}\frac{4}{3}-\frac{2}{3}x\ , & 0<x<1\\ 0\ , & \text{otherwise.}\end{cases}$$

(a) What is the probability of a pizza being delivered within half an hour of being ordered? (2 marks)

$P(X<0.5) = \int_0^{0.5} \left(\frac{4}{3}-\frac{2}{3}x\right)dx$ ✓

$= 0.5833$ ✓ $\left(\frac{7}{12}\right)$

(b) Calculate the mean delivery time to the nearest minute. (2 marks)

$E(X) = \int_0^1 x\left(\frac{4}{3}-\frac{2}{3}x\right)dx$ ✓

$= 0.4444$ i.e 27 minutes ✓

(c) Calculate the median delivery time in hours. (2 marks)

Solve $\int_0^m \frac{4}{3}-\frac{2}{3}x \, dx = 0.5$ ✓

$m = 0.4189$, $\cancel{3.5811}$

median $= 0.4189$ hours ✓

(d) Calculate the standard deviation of the delivery time. (2 marks)

$$Var(X) = \int_0^1 \left(\frac{4}{3} - \frac{2}{3}x\right)\left(x - \frac{4}{9}\right)^2 dx$$

$$= 0.0802 \qquad \left(\frac{\sqrt{26}}{18}\right) \quad ✓$$

$$\sigma = \sqrt{0.0802} \approx 0.2833 \text{ hrs or } 17 \text{ mins.} \quad ✓$$

(e) The pizza shop introduces new technology for its delivery drivers which decreases delivery times by 7.5%. Calculate the new mean and standard deviation for delivery times in minutes once the new technology has been implemented. (3 marks)

$$Y = 0.925X \quad ✓$$

$$\therefore E(Y) = 0.925E(X)$$

$$= 0.4111 \quad ✓$$

$$\sim 25 \text{ mins}$$

$$\sigma(Y) = |0.925| \times 0.2833$$

$$= 0.2621$$

$$\sim 16 \text{ mins} \quad ✓$$

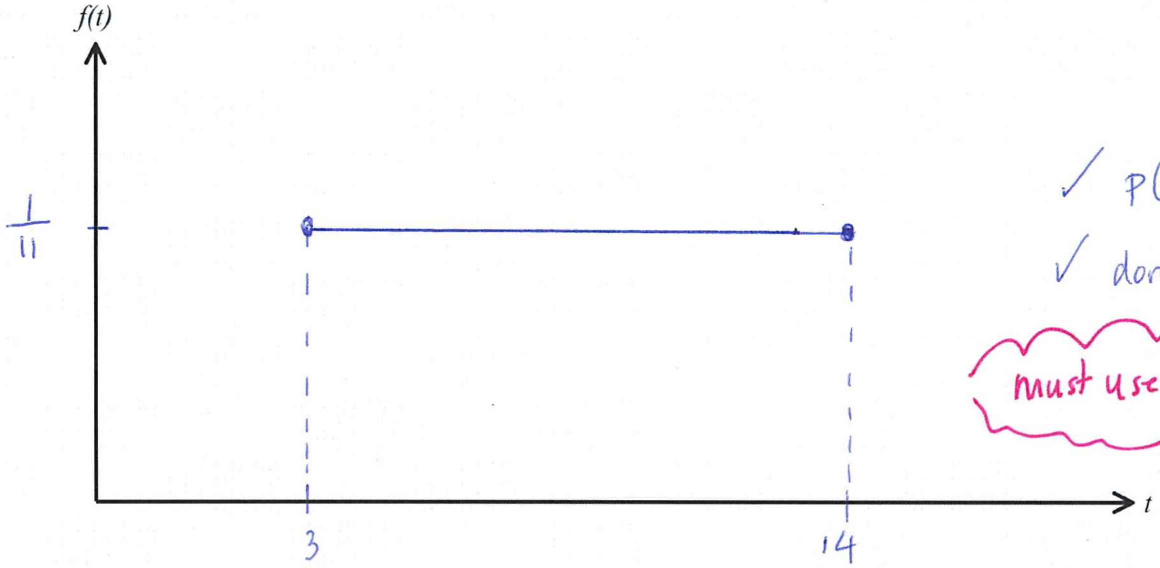
only penalise once for not changing to minutes in Q.

Question 8 **(7 marks)**

As part of a local arts festival, an artist plans to create an installation in which a concealed water cannon blasts a stream of water into the air for a few seconds at random intervals.

The lengths of the intervals between each firing of the cannon can be modelled by the uniformly distributed random variable T, where $3 \le t \le 14$ seconds.

(a) Sketch the probability density function $f(t)$ for the interval between each firing on the axes below. (2 marks)



✓ p(t)

✓ domain.

must use ruler

(b) Determine the probability that a randomly chosen interval between firings is

(i) at least seven seconds. (1 mark)

$P(7 \le t \le 14) = \frac{7}{11}$ ✓

(ii) at least six seconds given that it is less than ten seconds. (2 marks)

$P(t \ge 6 \mid t \le 10) = \frac{4/11}{7/11} = \frac{4}{7}$ ✓✓

(c) Determine the value of t for which $P(T < t) = P(T > 4t)$ (2 marks)

$\frac{t-3}{11} = \frac{14-4t}{11}$ ✓

$5t = 17$ $t = \frac{17}{5}$ ✓

END OF QUESTIONS